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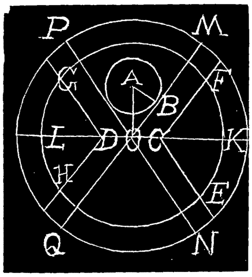
112. Proposed by **LON C. WALKER, A.M.**, Professor of Mathematics, Petaluma High School, Petaluma, Cal.

Two circles are drawn at random, both in magnitude and position, but so as to lie wholly upon the surface of a given circle. Show that the chance of their both resting on the same diameter of the given circle is $\frac{4}{\pi}(\log 2-5)$.

Solution by **G. B. M. ZERR, A. M., Ph. D.**, Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let A be the center of one of the circles, $AO=x$, $AB=y$, z=radius of other circle. With center O and radius $OK=r-z$ describe a circle. Draw MQ, NP through O tangent to circle center A.



Also draw CE, DH, CF, DG parallel to NP, MQ, respectively, at a distance z from them. If center of the second circle lies on either $CEKFC$ or $GDHLG$, both circles will not lie on the same diameter.

Area $CEKFC$+area $GDHLG$=

$$2\left[(r-z)^2\left[\cos^{-1}(y/x)-\sin^{-1}\left(\frac{z}{r-z}\right)\right]-z\sqrt{(r^2-2rz)}+\frac{yz^2}{\sqrt{(x^2-y^2)}}\right]=u.$$

The limits of z are 0 and $r\sqrt{(x^2-y^2)}/[x+\sqrt{(x^2-y^2)}]=z'$; of y, 0 and x, and 0 and $(r-x)$; of x, 0 and $\frac{1}{2}r$, and $\frac{1}{2}r$ and r.

$$\therefore p=1-\frac{\left[\int_{\frac{1}{2}r}^{r}\int_0^{r-x}2\pi x dx dy+\int_0^{\frac{1}{2}r}\int_0^{x}2\pi x dx dy\right]\int_0^{z}u dz}{\int_0^r\int_0^{r-x}\int_0^r\pi(r-z)^2.2\pi x dx dy dz}$$

$$=1-\frac{18}{\pi r^6}\left[\int_{\frac{1}{2}r}^{r}\int_0^{r-x}x dx dy+\int_0^{\frac{1}{2}r}\int_0^{x}x dx dy\right]\int_0^{z}u dz$$

$$=1-\frac{4}{5\pi r^3}\left[\int_{\frac{1}{2}r}^{r}\int_0^{r-x}\{15\cos^{-1}(y/x)-10+\frac{5}{y^5}[3x^2+3x\sqrt{(x^2-y^2)}-y^2]\right.$$

$$\times[x-\sqrt{(x^2-y^2)}]^3\}x dx dy+\int_0^{\frac{1}{2}r}\int_0^{x}\{15\cos^{-1}(y/x)-10$$

$$\left.+\frac{5}{y^5}[3x^2+3x\sqrt{(x^2-y^2)}-y^2][x-\sqrt{(x^2-y^2)}]^3\}x dx dy\right].$$

Let $y=x\sin\theta$.

Then $\theta=0$ and $\sin^{-1}\left[\frac{r-x}{x}\right]=\theta'$.

$$\therefore p=1-\frac{4}{5\pi r^3}\Big[\int_{\frac{1}{2}r}^{r}\int_0^{\theta}[15(\tfrac{1}{2}\pi-\theta)\cos\theta-10\cos\theta+10\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta-\tfrac{5}{2}\tan\tfrac{1}{2}\theta\sec^2\tfrac{1}{2}\theta]\times$$

$$x^2dxd\theta+\int_0^{\frac{1}{2}r}\int_0^{\frac{1}{2}\pi}[15(\tfrac{1}{2}\pi-\theta)\cos\theta-10\cos\theta+10\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta-\tfrac{5}{2}\tan\tfrac{1}{2}\theta\sec^2\tfrac{1}{2}\theta]x^2\,dxd\theta\ \Big]$$

$$=1-\frac{4}{5\pi r^3}\Big[\int_{\frac{1}{2}r}^{r}\Big(15(r-x)\cos^{-1}(\frac{r-x}{x})-10r-20\surd(2rx-r^2)$$

$$+\tfrac{65}{2}x-\frac{5x^2}{x+\surd(2rx-r^2)}\Big)xdx+\tfrac{15}{2}\int_0^{\frac{1}{2}r}x^2dx\Big]=\frac{4}{\pi}(8\log2-5).$$

Also solved with same result by the *PROPOSER*.

MISCELLANEOUS.

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107. **Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.**

The index of refraction of a medium varying inversely as the square root of the distance, prove that the path of a ray of light in the medium is a cycloid.

Solution by the PROPOSER.

Taking the axis of y in the given plane and that of x at right angles to the y axis, letting $\mu=k/\surd x$ be the index of refraction, and $p=dy/dx$, we have, by the usual theory, for the differential equation to the path

$$\frac{dp/dx}{1+p^2}=\frac{1}{\mu}\Big[\frac{d\mu}{dy}-\frac{d\mu}{dx}\frac{dy}{dx}\Big]....(1).\quad\text{We have }\frac{d\mu}{dx}=-\frac{k}{2x^{\frac{3}{2}}}....(2),$$

and (1) becomes $\dfrac{dp/dx}{1+p^2}=\dfrac{p}{2x}$, or $\dfrac{dp}{p(1+p^2)}=\dfrac{dx}{2x}....(3)$.

Integrating, $\log\dfrac{p}{\surd(1+p^2)}=\log\surd x+C....(4)$.

Let $p=b$, when $x=a$; then $C=\log\dfrac{b}{a\surd(1+b^2)}$,

and (4) becomes $p=\dfrac{dy}{dx}=\dfrac{xdx}{\surd[(a^2/b^2)(1+b^2)x-x^2]}....(5)$, the differential equation to a cycloid.

Also solved by *G. B. M. ZERR*, and *L. C. WALKER*.

108. **Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.**

To divide the arc of a cardioid into eight equal parts.